Solutions Total: 75 pts

Exam 1
Chapter 1

Answer the following questions. *You must show your work to receive full credit.* Indicate your final answer with a $\boxed{\text{box}}$.

1. (5 points) Create the truth table for implication; i.e. $p \to q$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

2. Consider the statement "If it is not raining or I have my umbrella, then I am not wet." Let

p :"It is raining," q :"I have my umbrella," and r :"I am wet."

(a) (5 points) Write the above statement using symbolic logic.

(b) (5 points) Suppose that r is true; i.e. I am wet. What compound statement involving p and q must be true? (You can use either symbolic logic or you can write out the statement in words.) **Hint:** Use the contrapositive.

(a) $(\neg p \vee q) \to (\neg r)$

(b) $r \to \neg(\neg p \vee q)$

$r \to \boxed{p \wedge \neg q}$

It is raining and I don't have my umbrella.

3. (10 points) Consider the following statements:

$p =$ "It is snowing today."
$q =$ "It is warmer than yesterday."
$r =$ " We will go skiing."
$s =$ "We will go to the mall.

Prove the following using a two-line proof (a proof table).

$$\left.\begin{array}{r} \neg p \wedge q \\ r \to p \\ \neg r \to s \end{array}\right\} \Rightarrow s$$

Statements	**Reasons**
1. $\neg p \wedge q$	given
2. $r \to p$	given
3. $\neg r \to s$	given
4. $\neg p$	simplification, 1
5. $\neg r$	modus tollens, 4, 2
6. s	modus ponens, 5, 3

4. The domain for this problem is some unspecified collection of numbers. Consider the predicate

$$P(x,y) = \text{"}x \text{ is greater than } y.\text{"}$$

(a) (4 points) Translate the following statement into predicate logic.

Every number has a number that is greater than it.

(b) (3 points) Negate your expression from part (a), and simplify it so that no quantifier or connective lies within the scope of a negation.

(c) (3 points) Translate your expression from part (b) into understandable English. Don't use variables in your English translation.

(a) $(\forall y)(\exists x)\, P(x,y)$

(b) $(\exists y)(\forall x)\neg P(x,y)$

(c) There is anumber such that no number is greater than it.

5. Recall that an integer a is even if there exists and integer k such that $a = 2k$. Let n_1 and n_2 be even integers.

(a) (3 points) Write n_1 and n_2 in terms of k_1 and k_2, respectively.

(b) (3 points) Write the product $n_1 n_2$ in terms of k_1 and k_2. Simplify your answer.

(c) (1 point) Is $n_1 n_2$ even or odd?

(a) $n_1 = 2k_1$, $n_2 = 2k_2$

(b) $n_1 n_2 = (2k_1)(2k_2) = 4k_1k_2$

(c) Even since $4k_1k_2 = 2(2k_1k_2)$ and $2k_1k_2$ is an integer.

6. (3 points) What is the difference between an axiom and a theorem?

An axiom is an assumption that is made; whereas a theorem is something that is proved from the axioms and definitions.

7. (10 points) Recall that for two integers x and y, we write $x|y$ whenever there exists an integer k such that $y = kx$. Prove the following statement using a direct proof:

Let a, b and c be integers. If $a|b$, then $a|(b \cdot c)$.

Let a,b and c be integers such that

$a|b$. Then $b = ka$ for some integer k.

Thus

$$bc = (ka)c = (kc)\cdot a$$

and hence $a|b\cdot c$ since kc is an integer.

8. (10 points) Recall that an integer a is odd if there exists and integer k such that $a = 2k + 1$. Also recall the definition of even on problem 6. Prove the following statement by contraposition:

Let x be an integer. If $x^2 + x + 1$ is even, then x is odd.

Let x be an even integer.

Then $x = 2k$ for some integer k.

Therefore

$$x^2 + x + 1 = (2k)^2 + 2k + 1 = 4k^2 + 2k + 1 = 2(2k^2 + k) + 1$$

is odd since $2k^2 + k$ is an integer.

9. (10 points) Recall that an angle is obtuse whenever it is greater than 90°. Use

Theorem 1 *The sum of the measures of the angles of any triangle (in Euclidean geometry) is equal to* 180°.

to give a proof by contradiction to the following statement:

A triangle cannot have more than one obtuse angle.

Let ΔABC be a triangle and suppose Δ has two angles $\angle A$ and $\angle B$ ~~each~~ which are obtuse.

Then $\angle A, \angle B > 90°$.

Therefore

$$\angle A + \angle B + \angle C > 90 + 90 + 0 = 180,$$

Contradicting Theorem 1.

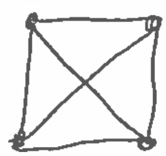
Extra Credit 1. (5 points)Recall the axiomatic system of the four-point geometry.

Undefined terms: point, line, is on
Axioms:

1. For every pair of distinct points x and y, there is a unique line l such that x is on l and y is on l.
2. Given a line l and a point x that is not on l, there is a unique line m such that x is on m and no point on l is also on m.
3. There are exactly four points.
4. It is impossible for three points to be on the same line.

Definition 4. A line l **passes through** x if x is on l.

How many distinct lines are there in the four-point geometry? **Hint:** Draw a model if it is helpful.



6 distinct lines

Extra Credit 2. (5 points) Let a be a logical statement. Is $a \to \neg a$ a contradiction?

a	$\neg a$	$a \to \neg a$
T	F	F
F	T	T

No!